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Divergence Management in Perturbative QCD

Shafeeq Rahman Thottoli[1] and Abdul Salih P P[2]

[1,2]Department of Physics, Aligarh Muslim University, Aligarh, UP-202002
E-mail: [1]shafeeqt@gmail.com, [2]ppsalih@gmail.com

Abstract—*The problem of divergence in quantum field theory is generally discussed. The divergences appearing in perturbative QCD and their management to get finite predictions are explained. Some popular methods for NLO QCD corrections that are applied to many physical processes are reviewed here.*

1. INTRODUCTION

In perturbative quantum field theory (QFT) contribution to perturbation series can be represented by Feynman diagrams and the corresponding analytical expressions can be obtained using Feynman rules. If a Feynman diagram contains a loop (or several loops) its expression contains integration over loop momenta. This integration can be divergent. Divergences arising from the large momentum (high energy) region of integral are called ultraviolet divergence (UV) which shows up where the momenta go to infinity. In the zero mass limits there exist divergences for small momenta (low energy) are called infrared (IR) divergences. There are two types of infrared divergences one which arises due to soft (i.e. small energy) massless particle like the photon in QED or the gluon in QCD is called soft divergence. The other one, called mass (collinear) divergences, arises when one massless field couples to another massless field or to itself. The origin of divergences depends on various factors of the propagators. In order to get finite result from the perturbative calculation we have to deal with infinities. UV divergence can be removed by a process called renormalization. There are various types of divergences in infrared sector and their management is done using different tricks. We are focusing on different methods used for the management of divergences arising in higher order perturbative calculations of QCD. We are reviewing some of the popular methods such as phase space slicing, subtraction method for carrying out such calculation.

2. DIVERGENCES IN QUANTUM FIELD THEORY

2.1 Ultraviolet divergence

Feynman diagram without any internal loop is termed as tree diagram; a diagram with one internal loop is one loop diagram and so on. If there are loops we have to integrate over loop momenta. Beyond tree level majority of them are divergent. As an example the momentum integration for a one loop Feynman diagram might be

$$I(p_1,p_2)=\int \frac{d^4k}{(2\pi)^4}\frac{\gamma^\nu(\not p_1+\not k)\gamma^\mu(\not p_2-\not k)\gamma_\nu}{k^2(p_1+k)^2(p_2-k)^2} \tag{1}$$

We can see that in case $k\to\infty$

$$I(p_1,p_2)\sim\int^\infty \frac{d^4k\,k^2}{k^6}\sim\int^\infty\frac{dk}{k} \tag{2}$$

These are infinite. These have to be regularized in order to get finite result. There are various possible regularization procedures. The most popular one is dimensional regularization[1][2]. In this method rather than changing the power of propagator in the momentum integration, we are allowing to change the dimension D in the integral to be an arbitrary complex number (D = 4 - 2v). Once integration is done and v tends to zero, the UV divergence appear as a pole in v and this can be removed by renormalization[2][3][4][5]

The philosophy behind renormalization is the following. The parameters we use in the starting theory are bare parameters, like couplings and masses, while in reality we measure the dressed parameters. We say that bare parameters already have infinities in them and the dressing process removes them resulting into finite and hence measurable physical quantities like coupling constants and masses which is finite.

3. INFRARED DIVERGENCES

In the zero mass limits there exist divergence for small momenta (low energy) which runs through internal loop or in phase space integral are called infrared (IR) divergences. Low energy occurring in the denominator can create the same problem that high energy occurring in the numerator does. If we take limit $k\to0$ of equation (1) become again will give infinite result.

$$I(p_1,p_2)\sim\int_0\frac{d^4k}{k^2\,p_1.k\,p_2.k}\sim\int_0\frac{dk}{k} \tag{3}$$

There are two types of infrared divergences. One when a massless particle emitted from an initial or final state particles

goes soft (i.e. its energy goes to zero) and is called soft divergences. They are encountered in both loop and phase space integral and found in the low energy region where the integration momenta go to zero. The other one is called collinear (mass) singularity which arises when one massless particle goes collinear to an external massless particle from which it has been emitted.

4. DIVERGENCE MANAGEMENT IN QCD

The usual divergences related to ultraviolet problems in any field theory appear in QCD are removed by the famous process of renormalization.

Then QCD have divergences related to infrared problems. There is a whole technology related to removal of such divergences. IR singularities are more complicate in QCD since gluon carry color charge.

It was Bloch and Eordsieck who first brought out the essential idea for the understanding of the infrared divergence problem in their famous paper[6] The idea was that in any practical scattering experiment involving charged particle it is impossible to specify complete final states of the system because of individual photons can be emitted with arbitrary small energies and always there is a possibility that some photons will escape from detection. The probability of only finite number of photons is precisely zero due to the IR divergence associated with the soft virtual photons. When the cross section is summed over all possible final state compatible with detector and including all undetected photons a non vanishing result is obtained. They showed that the observed cross section is very nearly the cross section that would be obtained if all radiative corrections are ignored. This is the well known cancellation between real and virtual infrared divergences.

It is common practice to use Bloch- Nordisieck trick [6] for the soft divergences. By adding the bremstrahlung process to the cross section with a virtual loop the overall cross section is soft finite.

The Block-Nordsieck trick does not work for collinear (mass) divergences. The standard approach to deal with these collinear divergences was developed by Lee and Nauenberg[7]. Their theorem states that collinear divergences will cancel after summing coherently over all physically indistinguishable states , known as Kinoshita – Lee – Nauenberg theorem (KLN theorem)[7][8]. This theorem states that all physical observables are finite.

Final state mass singularities cancel when summed over degenerate (experimentally indistinguishable) final states according to Kinoshitha- Lee- Nauenberg theorem [7] [8]. For tagged hadrons there is no final state sum and associated mass singularities are factorized into fragmentation functions. Similarly initial state singularities do not cancel because there is typically no sum over degenerate states they are removed by factorization[9][10]. Initial state singularities do not all cancel instead we absorb them into the definition of what are known as Parton Distribution Functions (PDF), which describe the probability of finding a certain type of parton within an incoming particle. The factorization theorom allows us to separate out the long range behaviour, including the collinear singularities and non-perturbative effects into these PDFs. The short distance behavior then has no initial state infrared divergences and can be calculated perturbatively.

5. PRACTICAL NLO CALCULATIONS

In next-to-leading order (NLO) perturbative QCD calculation, we have to consider real emission as well as virtual emission contributions. QCD correction to this process arises when a gluon is radiated off one of the outgoing quark which is real emission. A virtual gluon is exchanged from the quark-anti quark pair which is virtual the correction. IR divergence arises from both sectors. We have to manage these divergences to carry out finite calculation. The UV divergences appearing in NLO calculation can be taken care of with renormalization. We don't have to worry about UV divergences here.

NLO calculations combine virtual one-loop corrections with the real emission contributions from unresolved partons (quark and gluons). These two parts are usually computed separately and each is infrared divergent, only their sum is infrared finite.

The different phase space integrals which we have to perform to calculate a specific observable will generally contain different numbers of particles. This means that unless we are calculating fully inclusive quantities we must integrate the contributing terms separately over different phase space regions. In this case we are unable to take advantage of the cancellation of soft and collinear divergences at the integrand level. Consequently, this does not allow us to use a completely numerical approach for the calculation of these phase space integrals. NLO Monte Carlo programs evaluate both real emission and virtual diagrams and allow the simultaneous computation of many differential cross sections for the particular reaction considered. It requires that infrared singularities be eliminated before any numerical integration can be done. These are then integrated analytically, while the remaining finite regions are calculated numerically. There are various methods to do this. Two of these methods are the phase space slicing method[11] and subtraction method [12][13]

5.1 Phase space slicing

The basic idea of the phase space slicing method is to separate the phase space in two regions using the invariants $S_{ij} = 2P_i.P_j$, where the P_i are the momentum of the final state particles. The hard region is defined so that all the S_{ij} are bigger than a theoretical cut-off S_{min}. In this case, the calculation can be done numerically. The collinear and soft region is defined such that one or two S_{ij} are smaller than S_{min}. In this case, the calculation must be done analytically. If S_{min} is small enough, the soft and collinear approximation can be

used such that the integration in n dimensions is greatly simplifies. The soft approximation is generalized to the case where the particles involved are massive. In the collinear region, the mass regularizes the singularities and the calculation can be done numerically. The poles that remain after the integration over the soft and collinear region cancel with the corresponding poles of the virtual contributions.

5.2. Subtraction method

General approach of the subtraction method is shown in the papers[13][14][14][15]. The total NLO contribution to observables consists of a real piece $d\sigma^R$ (with m+1 partons in the final state) and a virtual piece $d\sigma^V$ (with m partons in the final state) the integration can then be rewritten as follows

$$\begin{aligned}\sigma_{NLO} &= \int_{m+1} d\sigma^R + \int_m d\sigma^V \\ &= \int_{m+1} (d\sigma^R - d\sigma^A) + \int_{m+1} d\sigma^A + \int_m d\sigma^V \\ &= \int_{m+1} (d\sigma^R - d\sigma^A) + \int_m \left(d\sigma^V + \int_1 d\sigma^A \right)\end{aligned}$$

Where $d\sigma^A$ is chosen in such a way that it has the same singular behavior as $d\sigma^R$. It acts as a local counter term and allowing first integral to be performed numerically. The first integral is finite by definition. So providing we are calculating an infrared safe observable, the final result of second integral must also be finite. This means that if we can calculate the single integral over $d\sigma^A$ analytically, we can once again remove the regulator from the second integral and calculate it numerically.

The crucial part of this process is that we are free to choose the form $d\sigma^A$ so that we are able to perform the single integral analytically. It is possible to define a method of generating $d\sigma^A$ for NLO process which is process independent. It is also possible for a suitable choice of $d\sigma^A$ to numerically integrate the virtual piece over the internal one-loop integral .Following these methods, it will make possible to use a completely numerical approach for any NLO observable.

The phase space slicing method of one cutoff developed[16] divides the phase space according to $s_{ij} = (p_i + p_j)^2 > ys_{12}$ where p_i and p_j label the momenta of partons *i* and *j*, and *y* is a small dimensionless parameter. The subtraction method taken together with factorization formula that interpolate between the soft and collinear approximations to the matrix elements is known as the dipole method[13]. In two cutoff phase space slicing method[17] two cutoff parameters are used to separate the region of phase space containing the soft and collinear singularities from the non- singular regions.

6. CONCLUSION

Perturbative quantum chromodynamics calculations are essential in understanding large momentum transfer hadronic cross sections. The need for increased precision for the theoretical calculation leads to the widespread use of NLO calculations. In these calculations we have to organize the soft and collinear singularity cancellation without loss of information in terms of observables. The divergence management is very important in the NLO calculation. Various methods are available to organize infrared divergences and may be useful for carrying out Monte-Carlo style calculations. We have discussed popular techniques available to resolve the physics out of these divergences. Phase –space slicing method and subtraction methods generally cancels the divergences. But in phase-space slicing we have to compromise on neglecting some portion of the phase-space. But Phase Phase-space slicing method with one cut off or two cut off and subtraction methods are popularly used in NLO QCD calculations.

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